22097303

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 1

Thursday 7 May 2009 (afternoon)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 7]*

Let $\boldsymbol{A} = \begin{pmatrix} 5 & 1 \\ 6 & 2 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 & -1 \\ -6 & 5 \end{pmatrix}$.

(a) (i) Find $\boldsymbol{AB}$.

(ii) Write down the inverse of $\boldsymbol{A}$. *[3 marks]*

Let $\boldsymbol{X} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$.

(b) Solve the matrix equation $\boldsymbol{AX} = \boldsymbol{C}$. *[4 marks]*

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2. *[Maximum mark: 6]*

The letters of the word PROBABILITY are written on 11 cards as shown below.

P	R	O	B	A	B	I	L	I	T	Y

Two cards are drawn at random without replacement.
Let A be the event the first card drawn is the letter A.
Let B be the event the second card drawn is the letter B.

(a) Find $\mathrm{P}(A)$. *[1 mark]*

(b) Find $\mathrm{P}(B|A)$. *[2 marks]*

(c) Find $\mathrm{P}(A \cap B)$. *[3 marks]*

3. *[Maximum mark: 6]*

Let $f(x) = \mathrm{e}^x \cos x$. Find the gradient of the normal to the curve of f at $x = \pi$.

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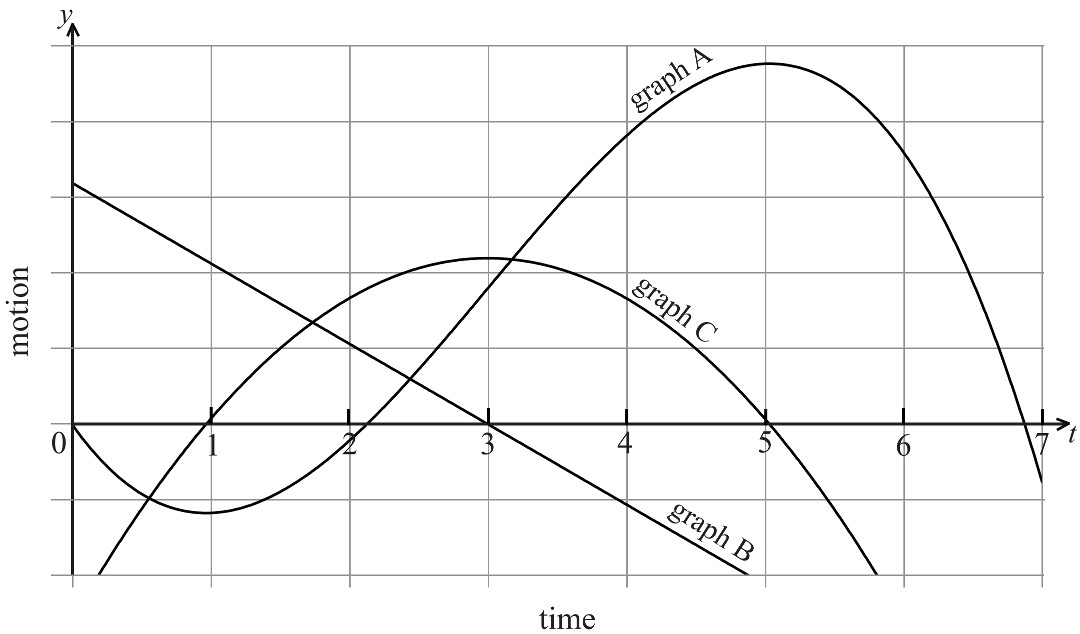
4. *[Maximum mark: 6]*

The following diagram shows the graphs of the **displacement**, **velocity** and **acceleration** of a moving object as functions of time, t.



(a) Complete the following table by noting which graph A, B or C corresponds to each function. *[4 marks]*

Function	**Graph**
displacement	
acceleration	

(b) Write down the value of t when the velocity is greatest. *[2 marks]*

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5. *[Maximum mark: 6]*

Let $f(x) = x^2$ and $g(x) = 2(x-1)^2$.

(a) The graph of g can be obtained from the graph of f using two transformations. Give a full geometric description of each of the two transformations. *[2 marks]*

(b) The graph of g is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ to give the graph of h.

The point $(-1, 1)$ on the graph of f is translated to the point P on the graph of h. Find the coordinates of P. *[4 marks]*

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6. *[Maximum mark: 7]*

Let $f(x) = e^{x+3}$.

(a) (i) Show that $f^{-1}(x) = \ln x - 3$.

(ii) Write down the domain of f^{-1}. *[3 marks]*

(b) Solve the equation $f^{-1}(x) = \ln\left(\frac{1}{x}\right)$. *[4 marks]*

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7. *[Maximum mark: 7]*

The graph of $y=\sqrt{x}$ between $x=0$ and $x=a$ is rotated 360° about the x-axis. The volume of the solid formed is 32π. Find the value of a.

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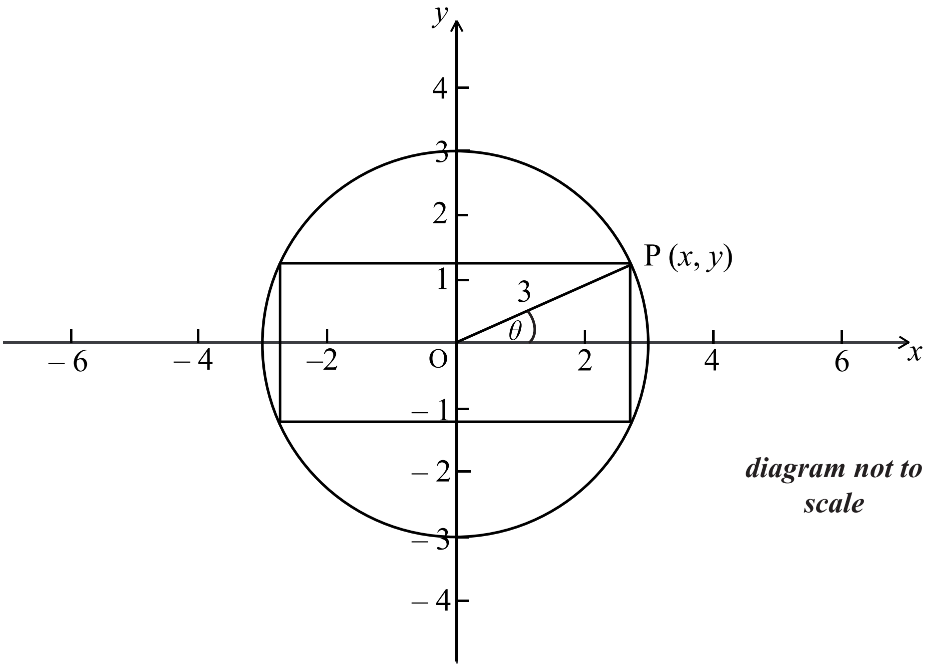
SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 13]*

A rectangle is inscribed in a circle of radius 3 cm and centre O, as shown below.



The point $\mathrm{P}(x, y)$ is a vertex of the rectangle and also lies on the circle. The angle between (OP) and the x-axis is θ radians, where $0 \le \theta \le \frac{\pi}{2}$.

(a) Write down an expression in terms of θ for

(i) x;

(ii) y. *[2 marks]*

Let the area of the rectangle be A.

(b) Show that $A = 18 \sin 2\theta$. *[3 marks]*

(c) (i) Find $\frac{\mathrm{d}A}{\mathrm{d}\theta}$.

(ii) Hence, find the exact value of θ which maximizes the area of the rectangle.

(iii) Use the second derivative to justify that this value of θ does give a maximum. *[8 marks]*

*Do **NOT** write on this page.*

9. *[Maximum mark: 16]*

The vertices of the triangle PQR are defined by the position vectors

$$\overrightarrow{OP} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}, \ \overrightarrow{OQ} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \text{ and } \overrightarrow{OR} = \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}.$$

(a) Find

(i) $\overrightarrow{PQ}$;

(ii) $\overrightarrow{PR}$. *[3 marks]*

(b) Show that $\cos R\hat{P}Q = \frac{1}{2}$. *[7 marks]*

(c) (i) Find $\sin R\hat{P}Q$.

(ii) Hence, find the area of triangle PQR, giving your answer in the form $a\sqrt{3}$. *[6 marks]*

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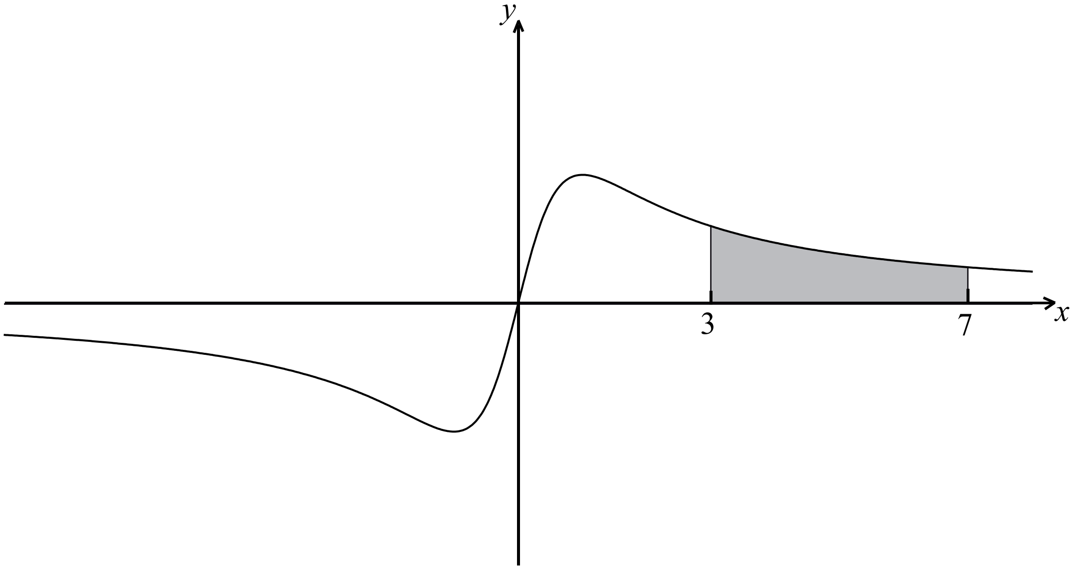
10. *[Maximum mark: 16]*

Let $f(x)=\frac{ax}{x^2+1}$, $-8\le x\le 8$, $a\in\mathbb{R}$. The graph of f is shown below.



The region between $x=3$ and $x=7$ is shaded.

(a) Show that $f(-x)=-f(x)$. *[2 marks]*

(b) Given that $f''(x)=\frac{2ax(x^2-3)}{(x^2+1)^3}$, find the coordinates of all points of inflexion. *[7 marks]*

(c) It is given that $\int f(x)\,\mathrm{d}x=\frac{a}{2}\ln(x^2+1)+C$.

(i) Find the area of the shaded region, giving your answer in the form $p\ln q$.

(ii) Find the value of $\int_4^8 2f(x-1)\,\mathrm{d}x$. *[7 marks]*